Peak to Average Power Ratio Reduction Using Zadoff-Chu Transformation Pre-Coding Technique

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Abstract
Orthogonal Frequency Division Multiplexing (OFDM) is a technique used for modern high speed wireless communication networks. It not only provides high data rate but also offers high spectral efficiency and for in-band and out-band noise immunity. But OFDM networks suffers from the major problem associated with its high peak to average power ratio (PAPR). High value of PAPR drives the power amplifier to operate in non-linear region and hence results in intermodulation distortion. This drawback can be overcome by using PAPR reduction techniques. In this paper pre-coding technique has been used to get low value of PAPR.

Keywords–Orthogonal Frequency Division Multiplexing (OFDM), Peak to Average Power Ratio (PAPR), Power Amplifier, Walsh-Hadamard Transform (WHT), Zadoff-Chu Transformation (ZCT)

Introduction
An efficient technique for the effective use of wireless communication is Orthogonal Frequency Division Multiplexing (OFDM), is one of multi-carrier modulation (MCM) techniques, multipath delay spread tolerance, offer a considerable high spectral efficiency, immunity to the frequency selective fading channels and power efficiency. Therefore, OFDM has been chosen for high data rate communications and has been widely deployed in many wireless communication standards such as Digital Video Broadcasting (DVB) and based mobile worldwide interoperability for microwave access based on OFDM access technology. The foremost problem is high Peak-to-Average Power Ratio (PAPR) of transmitted OFDM signals. Thus, it is important and necessary to research on the characteristics of the PAPR including its distribution and reduction in OFDM systems, in order to utilize the technical features of the OFDM. The distribution of PAPR, which bears stochastic characteristics in OFDM systems, often can be expressed in terms of Complementary Cumulative Distribution Function (CCDF). An effective PAPR reduction technique should be given the best tradeoff between the capacity of PAPR reduction and transmission power, data rate loss, implementation complexity and Bit-Error-Ratio (BER) performance etc. [1-3] Mostly the techniques of reducing PAPR distributed the wave shape of signal that results in inter-carrier-interference or out of band noise or increase in BER and most of them reduce PAPR at the expense of bandwidth. Similarly, the coding techniques do not affect the signal rather they transform or code the data using different algorithm to minimize PAPR. These algorithms that are used, increase the computational complexity and demand more time for more complex that is undesirable for high speed communication. In this paper, firstly we investigate the distribution of PAPR based on the characteristics of the OFDM signals. Then, we analyze five typical
techniques of PAPR reduction and propose the criteria of PAPR reduction in OFDM systems using ZCT and Walsh codes.

Pre-Coding Techniques for OFDM

In pre-coding techniques, the data is transformed or coded before transmission that saves time and ensure high speed transmission. The block diagram of Pre-coded OFDMA is shown in Fig.2.

![Fig. 1. Block diagram for Pre-coded OFDMA system.](image)

The pre-coding part is shown with block “P” in the Fig. 1. The pre-coding is usually done after IFFT on the transmitter side and inverse pre-coding is done before FFT on receiver side [5]. The pre-coding and inverse pre-coding cancel the effect of each other and provides a mechanism to reduce PAPR by reshaping the data. The Pre-coding technique may be referred to an algorithm that helps to reduce PAPR. The pre-coding techniques like WHT, DHT, ZCT and DFT are briefly discussed below:

(I) Walsh-Hadamard Transform (WHT)

This is the simplest and linear transformation that is applied as butterfly structure of FFT. WHT does not increase the complexity of the system. Mathematically, WHT can be expressed as:

\[ H_1 = [1] \]

\[ H_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \]

\[ H_N = \frac{1}{\sqrt{2N}} \begin{bmatrix} H_N & H_N^{-1} \\ H_N & H_N^{-1} \end{bmatrix} \]

Where \( H_N^{-1} \) is the binary complement of \( H_N \), while this method helps to mitigate PAPR but it is not as efficient as the later techniques which are described below:
(II) Discrete Hartley Transformation (DHT)

The DHT is also a linear transformation of data. In DHT, N real numbers \(x_0, x_1, \ldots, x_{N-1}\) are transformed into real numbers \(H_0, H_1, \ldots, H_{N-1}\). According to, the N-point DHT can be defined as follows:

\[
H_k = \sum_{n=0}^{N-1} x_n \left[ \cos \left( \frac{2\pi nk}{N} \right) + \sin \left( \frac{2\pi nk}{N} \right) \right]
\]

(1)

The pre-coding based matrix can be constructed as follows:

\[
A = \begin{bmatrix}
a_{00} & a_{01} & \cdots & a_{0(L-1)} \\
a_{10} & a_{11} & \cdots & a_{1(L-1)} \\
\vdots & \vdots & \ddots & \vdots \\
a_{(L-1)0} & a_{(L-1)1} & \cdots & a_{(L-1)(L-1)}
\end{bmatrix}
\]

(2)

A is pre-coding matrix of size \(L \times L\) shown in above equation, \(m\) and \(l\) are integers from 0 to \(L-1\). The DHT is also invertible transform which allows us to recover \(x_n\) from \(H_k\) and inverse can be obtained by simply multiplying DHT of \(H_k\) by \(\frac{1}{N}\).

(III) Zadoff-Chu Transformation (ZCT)

The Zadoff-chu sequences have optimum correlation properties with ideal periodic autocorrelation and constant magnitude. According to ZC sequences of length \(L\) can be defined as:

\[
a_n = \begin{cases} 
\frac{\sin(\pi n/2)}{\sin(\pi n/L)} & \text{for } L \text{ even} \\
\frac{\sin(\pi n(\frac{L+1}{2})}{\sin(\pi n/L)} & \text{for } L \text{ odd}
\end{cases}
\]

(3)

Where \(L\) is any integer. The by restructuring the Zadoff-chu sequence by the ZC transformation matrix (ZCMT) for can be written as:

\[
A = \begin{bmatrix}
a_{00} & a_{01} & \cdots & a_{0(L-1)} \\
a_{10} & a_{11} & \cdots & a_{1(L-1)} \\
\vdots & \vdots & \ddots & \vdots \\
a_{(L-1)0} & a_{(L-1)1} & \cdots & a_{(L-1)(L-1)}
\end{bmatrix}
\]

(4)
Here is the row variable and \( l \) the column variable. In other words, the \( L^2 \) point long ZC sequence fills the kernel of the matrix transform row-wise. ZCMT is implemented to the uplink signal and by simulation it is shown that the ZCMT reduces PAPR significantly as compared to conventional WHT pre-coding based technique. This technique is independent of the nature of input signal and also it can offer substantial gain in fading multipath channels due to frequency variation in communication channels.

(IV) Discrete Fourier Transform (DFT)

In DFT pre-coding technique, the size of pre-coder is same as it is of IFFT used to combine separate signals and the resultant signal need only a single carrier because DFT and IDFT cancel each other. In this way, the system becomes just like a single-carrier system but its PAPR is improved. The Pre-coding matrix \( P \) of dimension is used before the IFFT to reduce the PAPR. The pre-coding matrix \( P \) can be written as:

\[
P = \begin{bmatrix}
p_{00} & p_{01} & \cdots & p_{0(L-1)} \\
p_{10} & p_{11} & \cdots & p_{1(L-1)} \\
\vdots & \vdots & \ddots & \vdots \\
p_{(L-1)0} & p_{(L-1)1} & \cdots & p_{(L-1)(L-1)}
\end{bmatrix}
\]  

(5)

The signal with subcarriers can be written as complex base band OFDM as follows:

\[
x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} P.X_k e^{j2\pi k}\Delta f t, 0 \leq t \leq NT
\]  

(6)

The vector signal with subcarriers modulated OFDM can be written as:

\[
x_N = IFFT \{ P.X_N \}
\]  

(7)

The PAPR of OFDM signal can be written as:

\[
PAPR = \frac{\max|X(t)|^2}{E[|x(t)|^2]}
\]  

(8)

The DFT and IDFT sequence of length \( N \) can be written as:

\[
X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk}, \quad k = 0, 1, \cdots, N - 1
\]  

(9)

\[
x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi nk}, \quad k = 0, 1, \cdots, N - 1
\]  

(10)

Where, \( p_{mn} = e^{j\frac{2\pi mn}{N}} \)  

Simulation Results
MATLAB has been used as a simulation platform. To evaluate the PAPR performance of the ZCT precoding OFDM system, its performance has been compared with the performance of Walsh Hadamard codes. To show the effect of these techniques, an OFDM system has been considered with M-QAM modulation. Results have been compared with M = 64 and 256. Figure 2 shows the CCDF comparisons of PAPR of conventional OFDM, Walsh OFDM and ZCT OFDM system M=64 modulation.

Fig. 2. CCDF comparisons for M=64

Fig.3. CCDF comparisons for M=64
Figure 3 shows the CCDF comparisons of PAPR of conventional OFDM, Walsh OFDM and ZCT OFDM system M=256 modulation. Results show that both for M=64 and 256, ZCT based PAPR reduction technique perform better as compared to Walsh technique.

**Conclusion**

Conventional PAPR reduction techniques are less efficient and in most of these techniques a tradeoff between bandwidth and PAPR reduction is required. Coding techniques can be explored for PAPR reduction but these requires computational complexity. So, in this paper, a pre-coding technique has been used for PAPR reduction of an OFDM system. Simulation results show the effectiveness of the proposed technique in PAPR reduction.

**References**